On the Extraction of a Common Persistent Component from Several Volatility Indicators

Fabrizio Cipollini, Giampiero M. Gallo

Abstract Measures of financial volatility exhibit clustering and persistence and can be jointly modeled as the element by element product of a vector of conditionally autoregressive scale factors and a multivariate i.i.d. innovation process (vector Multiplicative Error Model – vMEM). Since similar profiles are shared across measures, a restricted vMEM decomposes the conditional expected volatility into the sum of a common (persistent) component and a vector of measure specific components. With data on absolute returns, realized kernel volatility and daily range for the Dow Jones index, we show that indeed such a common component exists with the desired properties. The transitory components happen to have different features across volatility measures.

Key words: Volatility, (vector) Multiplicative Error Models, Long/Short Run Decomposition, GARCH, GMM, Penalized Estimation.

1 Introduction

In financial time series analysis, a centerpiece is dedicated to volatility measurement and modeling/forecasting. The dynamic interdependence among several indicators of volatility (absolute returns, daily range and realized volatility) was performed in [6] as a stack of univariate MEMs. Extending this analysis, the idea that a gain in estimation efficiency and a more reliable interpretation of the significance of the links can be obtained with a vector MEM (or vMEM) approach is pursued by [3, 4].

We motivate our new model, labeled *Additive Common Component vMEM* or ACC-vMEM, starting from the matching patterns exhibited by several indicators of

Fabrizio Cipollini
Dip. di Statistica "G. Parenti", Università di Firenze. e-mail: `cipollini@ds.unifi.it`

Giampiero M. Gallo
Dip. di Statistica "G. Parenti", Università di Firenze. e-mail: `gallog@ds.unifi.it`

volatility, translating the idea of similar high persistence or long–term evolution in each series into a common component. Relative to the vMEM, the essential feature of the new model is the structure of the conditional mean, that is decomposed into the sum of a common (persistent) component plus a vector of idiosyncratic individual components. The main advantage is an enhanced interpretation of the dynamics: scenario analysis can be built on different assumptions relative to the evolution of the two components; relatively parsimonious formulations can capture fairly rich dynamic patterns, retaining, as with any vMEM, the ability of multistep forecasts. The model, whose reduced form can be traced to a more complex vMEM formulation with constrained parameters, has connections with the Composite–MEM considered in [2], whose characteristics have proved to adequately capture the dynamics of realized volatilities of single assets. The seminal idea, however, should be traced back to the univariate GARCH model proposed in [7], in which the dynamics of the conditional variance is decomposed into two additive components, one labeled as permanent in view of its higher persistence, and the other one as transitory.

Given the semiparametric specification of the model, inferences can be obtained via GMM (Generalized Method of Moments), based on the conditional mean and variance expressions following from the model definition, according to [4] and extended by [5] to a penalized version to accommodate empirical applications possibly affected by identification issues and/or highly collinear effects.

We estimate our model on three indicators of volatility (absolute returns, realized volatility and daily range) extracting the common persistent component from data on the Dow Jones index. The results show that the common dynamics is well supported by the data and captures a high degree of the total behavior of the series; the transitory components show little persistence; finally, in estimating conditional expectations, we find the highest accuracy for realized volatility, followed by the daily range and by absolute returns.

2 The vMEM and the Additive Common Component vMEM

Let $\{\boldsymbol{x}_t\}$ a discrete time process with components defined on $[0,+\infty)^K$. In the vMEM (vector Multiplicative Error Model [3, 4]), $\boldsymbol{x}_t$ is structured as

$$\boldsymbol{x}_t = \boldsymbol{\mu}_t \odot \boldsymbol{\varepsilon}_t \tag{1}$$

where, conditionally on the information $\mathscr{F}_{t-1}$, $\boldsymbol{\mu}_t$ is deterministic,

$$\boldsymbol{\mu}_t = \mu(\boldsymbol{\theta}, \mathscr{F}_{t-1}) \tag{2}$$

and $\boldsymbol{\varepsilon}_t$ is stochastic, with pdf (probability density function) defined over a $[0,+\infty)^K$ support and such that

$$\boldsymbol{\varepsilon}_t | \mathscr{F}_{t-1} \sim D^+(\mathbb{1}, \boldsymbol{\Sigma}). \tag{3}$$

The previous assumptions on $\boldsymbol{\mu}_t$ and $\boldsymbol{\varepsilon}_t$ give

$$E(\boldsymbol{x}_t|\mathscr{F}_{t-1}) = \boldsymbol{\mu}_t \qquad V(\boldsymbol{x}_t|\mathscr{F}_{t-1}) = \boldsymbol{\mu}_t\boldsymbol{\mu}_t' \odot \boldsymbol{\Sigma} = \mathrm{diag}(\boldsymbol{\mu}_t)\boldsymbol{\Sigma}\,\mathrm{diag}(\boldsymbol{\mu}_t), \tag{4}$$

where the latter is a positive definite matrix by construction.

Assuming that $\boldsymbol{x}_t$ is mean-stationary with $E(\boldsymbol{x}_t) = E(\boldsymbol{\mu}_t) = \boldsymbol{\mu}$, $\boldsymbol{\mu}_t$ can be specified as

$$\boldsymbol{\mu}_t = \boldsymbol{\mu} + \boldsymbol{\xi}_t \tag{5}$$

$$\boldsymbol{\xi}_t = \boldsymbol{\beta}_1^*\boldsymbol{\xi}_{t-1} + \boldsymbol{\alpha}_1\boldsymbol{v}_{t-1} + \boldsymbol{\gamma}_1\boldsymbol{v}_{t-1}^{(-)} \tag{6}$$

$$\boldsymbol{v}_t = \boldsymbol{x}_t - \boldsymbol{\mu}_t \qquad \boldsymbol{v}_t^{(-)} = \boldsymbol{x}_t^{(-)} - \boldsymbol{\mu}_t/2, \tag{7}$$

where the vector $\boldsymbol{x}_t^{(-)}$ aims at capturing asymmetric effects associated with the sign of an observed variable and is usually structured as $x_{t,j}^{(-)} = x_{t,j}I_{t,j}^{(-)}$, where $I_{t,j}^{(-)}$ denotes the indicator of a negative value of the signed variable (see [4] for more details). Further lags could be added, but we do not consider them here. From a practical point of view, the $\boldsymbol{\mu}_t$ defined by (5)-(6)-(7) constitutes a trivial reparameterization of the usual $\boldsymbol{\mu}_t$-equation in [4]. However, it has the merit of representing the dynamics of the process being driven by a zero mean, stationary component, $\boldsymbol{\xi}_t$, that moves around the *unconditional* average level $\boldsymbol{\mu}$. Depending on the context, further meaningful components, similar to $\boldsymbol{\xi}_t$, can be added and/or a time–varying rather than a fixed level $\boldsymbol{\mu}$, can be taken into account.

In this spirit, one can note as non-negative financial time series tend to show frequently very similar patterns over the sample of observation (see the *Data* panel of Figure 1 for an example), conveying the idea of a single underlying driving force. Accordingly, we propose a new formulation of the vMEM by changing the structure of $\boldsymbol{\mu}_t$ as follows: the fixed $\boldsymbol{\mu}$ is replaced by a time-varying component driven by a scalar common factor, η_t; $\boldsymbol{\xi}_t$ is (ideally) structured as a vector of specific elements, i.e. each $\xi_{t,j}$ depends on its own past values only. The implicit assumption is that the common component is able to capture adequately the main part of the cross-dependence. More explicitly, this new *Additive Common Component* vMEM (or ACC-vMEM) has $\boldsymbol{\mu}_t$ defined by

$$\boldsymbol{\mu}_t = \boldsymbol{\mu} + \boldsymbol{\psi}\eta_t \tag{8}$$

$$\boldsymbol{\xi}_t = \boldsymbol{\beta}_1^{(\xi)*}\boldsymbol{\xi}_{t-1} + \boldsymbol{\alpha}_1^{(\xi)}\boldsymbol{v}_{t-1} + \boldsymbol{\gamma}_1^{(\xi)}\boldsymbol{v}_{t-1}^{(-)} \tag{9}$$

$$\eta_t = \beta_1^{(\eta)*}\eta_{t-1} + \boldsymbol{\alpha}_1^{(\eta)\prime}\boldsymbol{v}_{t-1} + \boldsymbol{\gamma}_1^{(\eta)\prime}\boldsymbol{v}_{t-1}^{(-)} \tag{10}$$

together with (7) and, in order to make the model identified, $\boldsymbol{\psi}'\mathbb{1} = K$. As detailed in [5], this ACC-vMEM has connections with other models: for $K = 1$ it collapses into the Composite–MEM by [2] that, at the time, has close similarities with the component GARCH of [7]; for $\boldsymbol{\alpha}_1^{(\eta)} = \boldsymbol{\gamma}_1^{(\eta)} = \mathbf{0}$ it degenerates into a vMEM without common components; in general, it can be seen as the reduced form of a more complex vMEM formulation.

3 Empirical Application: Common Dynamics in Volatility

We adopt the ACC-vMEM for modeling the joint dynamics of absolute returns ($|ret|$), realized kernel volatility (*rkv*) and high-low range (*hl*) for the DJ30 (Dow Jones Industrials) index in the period February 2001 – February 2009 ($T = 2009$ observations). Realized kernel volatilities are computed from tick by tick data according to [1] and taken from the Oxford Man Institute (OMI) *Realized Library* (`http://realized.oxford-man.ox.ac.uk/`), that uses data from *Reuters DataScope Tick History*. Returns and daily ranges are computed using the daily highs and lows downloaded from Datastream. All measures are expressed in annualized percentage terms.

The aim of the analysis is to illustrate the separate contribution, to the overall dynamics, of the common and measure–specific transitory components, investigating their relative importance across the three indicators. In order to gain some insight into the changes provided by the Common Component, different models are compared: an ACC-vMEM in which $\boldsymbol{\gamma}_1^{(\eta)} = \mathbf{0}$ and all $\boldsymbol{\alpha}_1^{(\xi)}$, $\boldsymbol{\gamma}_1^{(\xi)}$, $\boldsymbol{\beta}_1^{(\xi)}$ coefficient matrices are diagonal (label *c-diag*); a model obtained adding to *c-diag* the asymmetric effects in the permanent component and a full $\boldsymbol{\alpha}_1^{(\xi)}$ matrix (label *c-full*); two corresponding vMEM's, labeled *diag* and *full* respectively, obtained from the previous ones by removing the common component. All models are estimated by penalized GMM (cf. [5]) using relatively small penalties.

The time series, shown in the top panel of figure (1), highlight the strongly similar pattern of the long term evolution of the three indicators, with an initial period of (relatively) high volatility, followed by a long period of low volatility (between 2004 and the first half of 2007), in turn followed by a progressive increase up to the burst at the end of 2008. Around this long term pattern, *rkv* appears considerably less noisy than $|ret|$, with *hl* in an intermediate position.

The parameter estimates (Table 1) show a high persistence of the common component throughout, substantially in line with the univariate analyses in [2] and [7]. The main contribution to this component is provided by the *rkv* innovations ($\alpha_{2,1}^{(\eta)}$ is highly significant), and this seems consistent with the smoother evolution of this indicator relative to the remaining ones. The impact of the $|ret|$ and *hl* innovations on the common component is by far less important and loss its significance in the most complex ACC-vMEM. Judging on the size of the common component's loadings, η_t is usually lowered when transferring into *rkv* (ψ_2 significantly smaller than 1 for all indices), while it is amplified when entering into $|ret|$ and *hl* (ψ_1, ψ_3 significantly higher than 1). The estimated parameters also point to the fact that the persistent component captures an important part of the persistence of the specific series: while the vMEMs have a fairly high $\boldsymbol{\beta}_1^{(\xi)*}$'s coefficients, the corresponding ACC models show a sharp decrease in their value. Note also as the $\boldsymbol{\alpha}_1^{(\xi)}$ estimates tend to change between the vMEM and the ACC–vMEM with a corresponding structure of the short term component. Moreover, a comparison of the last two columns reveals as the asymmetric effects seem to have a larger impact on the permanent than on the short term component.

Table 1 Parameter estimates (est.) and corresponding standard erors (s.e.) for different specifications of the vMEM for the DJ30 ticker (20010201 – 20090227): with all diagonal coefficient matrices (*diag*), with full $\boldsymbol{\alpha}^{(\xi)}$ (*full*), Common Component with all diagonal coefficient matrices in the short term component (*c-diag*), Common Component with full $\boldsymbol{\alpha}^{(\xi)}$ (*c-full*).

	diag		*full*		*c-diag*		*c-full*	
parameter	est.	s.e.	est.	s.e.	est.	s.e.	est.	s.e.
μ_1	12.0038	0.8291	11.7688	0.7347	11.9113	1.1366	13.4386	1.1123
μ_2	11.5170	0.5942	11.4914	0.6260	11.7555	1.0109	13.0744	0.9961
μ_3	13.0198	0.5708	12.9887	0.7146	13.2438	1.1651	14.8029	1.1328
$\beta_{1,1,1}^{(\xi)*}$	0.9832	0.0034	0.9653	0.0054	0.4171	0.1220	0.3869	0.0827
$\alpha_{1,1,1}^{(\xi)}$	0.0075	0.0084	-0.0131	0.0117	-0.1182	0.0189	-0.0827	0.0302
$\alpha_{1,2,1}^{(\xi)}$			0.3197	0.0426			0.3926	0.0936
$\alpha_{1,3,1}^{(\xi)}$			0.0270	0.0245			-0.2191	0.0656
$\gamma_{1,1,1}^{(\xi)}$	0.0775	0.0104	0.0408	0.0092	0.0681	0.0215	0.0065	0.0207
$\beta_{2,2,1}^{(\xi)*}$	0.9701	0.0045	0.9554	0.0063	0.4413	0.0914	0.4591	0.0813
$\alpha_{2,1,1}^{(\xi)}$			0.0128	0.0090			-0.0086	0.0120
$\alpha_{2,2,1}^{(\xi)}$	0.2287	0.0140	0.3036	0.0278	0.0516	0.0240	0.1464	0.0387
$\alpha_{2,3,1}^{(\xi)}$			0.0638	0.0208			0.0233	0.0271
$\gamma_{2,2,1}^{(\xi)}$	0.0746	0.0077	0.0385	0.0078	0.0662	0.0101	0.0053	0.0122
$\beta_{3,3,1}^{(\xi)*}$	0.9767	0.0030	0.9625	0.0050	0.4416	0.0773	0.2791	0.0819
$\alpha_{3,1,1}^{(\xi)}$			0.0053	0.0093			-0.0289	0.0182
$\alpha_{3,2,1}^{(\xi)}$			0.2988	0.0305			0.1921	0.0578
$\alpha_{3,3,1}^{(\xi)}$	0.0487	0.0082	0.0399	0.0218	-0.1274	0.0165	-0.1464	0.0412
$\gamma_{3,3,1}^{(\xi)}$	0.0903	0.0072	0.0467	0.0069	0.0914	0.0136	0.0364	0.0144
ψ_1					1.0172	0.0139	1.0217	0.0092
ψ_2					0.9213	0.0115	0.9139	0.0071
ψ_3					1.0615	0.0112	1.0644	0.0071
$\beta_1^{(\eta)*}$					0.9789	0.0056	0.9823	0.0026
$\alpha_{1,1}^{(\eta)}$					0.0161	0.0087	-0.0154	0.0117
$\alpha_{2,1}^{(\eta)}$					0.2859	0.0321	0.1717	0.0307
$\alpha_{3,1}^{(\eta)}$					0.0470	0.0201	-0.0036	0.0261
$\gamma_{1,1}^{(\eta)}$							0.0669	0.0163
$\gamma_{2,1}^{(\eta)}$							-0.0860	0.0319
$\gamma_{3,1}^{(\eta)}$							0.1005	0.0361
σ_1	0.8333		0.8295		0.8192		0.7991	
σ_2	0.2436		0.2439		0.2447		0.2330	
σ_3	0.3994		0.4004		0.4006		0.3849	
$R_{1,2}$	0.1847		0.2435		0.2418		0.2190	
$R_{1,3}$	0.7284		0.7322		0.7317		0.7228	
$R_{2,3}$	0.5678		0.6179		0.6209		0.6024	

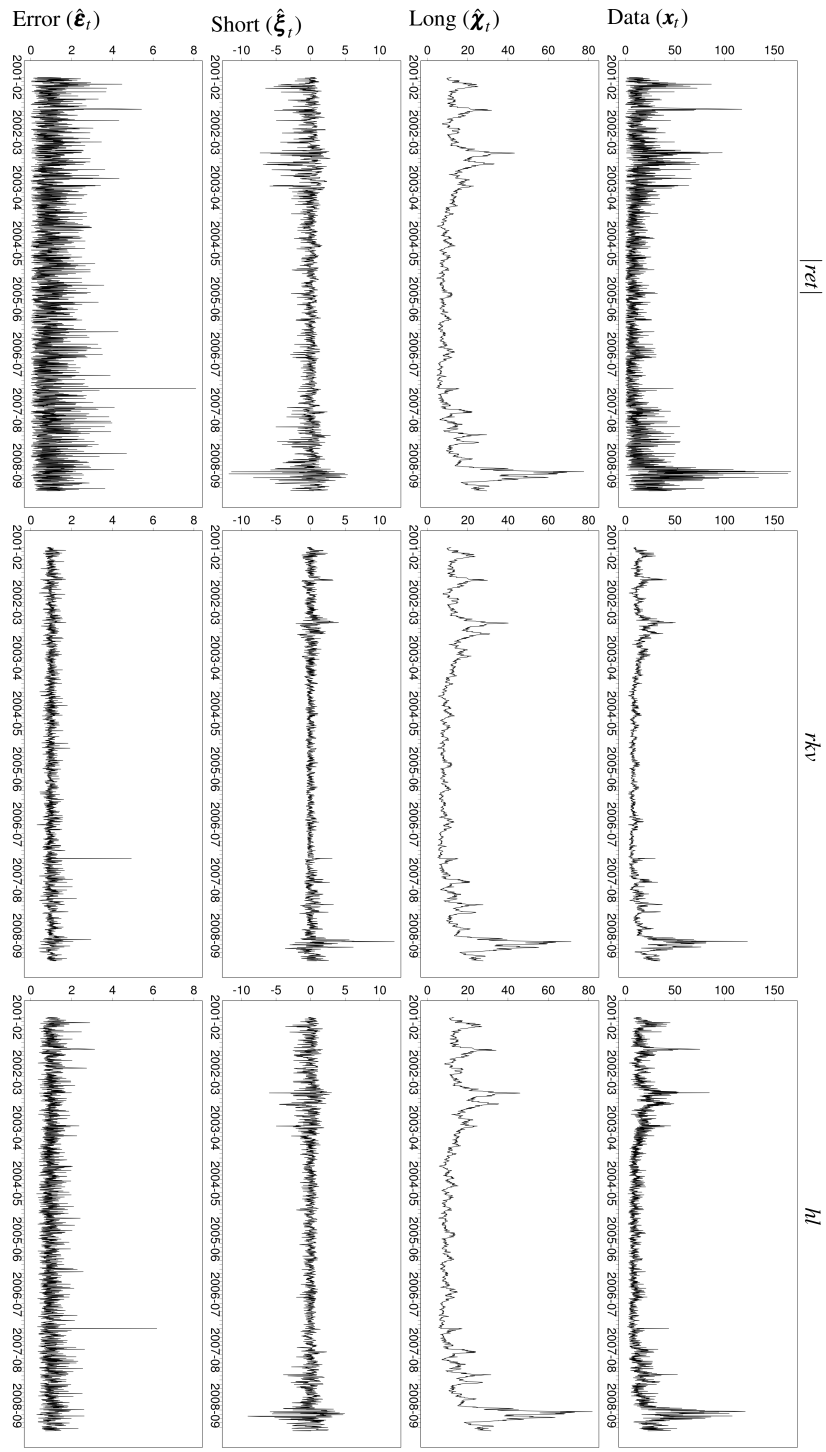

Fig. 1 Graphs of data, estimated persistent component ($\hat{\boldsymbol{\chi}}_t = \hat{\boldsymbol{\mu}} + \hat{\boldsymbol{\psi}}\hat{\eta}_t$), short term component ($\hat{\boldsymbol{\xi}}_t$) and error ($\hat{\boldsymbol{\varepsilon}}_t$), of the ACC-vMEM with diagonal coefficient matrices in the short term component, for the DJ30 ticker (20010201 – 20090227).

Table 2 Ljung-Box statistics for different specifications of the vMEM for the DJ30 ticker (20010201 – 20090227): with all diagonal coefficient matrices (*diag*), with full $\boldsymbol{\alpha}^{(\xi)}$ (*full*), Common Component with all diagonal coefficient matrices in the short term component (*c-diag*), Common Component with full $\boldsymbol{\alpha}^{(\xi)}$ (*c-full*).

lag	*diag*	*full*	*c-diag*	*c-full*
12	9.0624E-28	1.5444E-09	3.0464E-06	1.0366E-02
22	4.8892E-18	3.0500E-05	4.9878E-03	2.2769E-01
32	1.9827E-13	5.8010E-04	4.4802E-02	5.1617E-01

Table 3 Diebold-Mariano statistics between couples of different specifications of the vMEM for the DJ30 ticker (20010201 – 20090227): with all diagonal coefficient matrices (*diag*), with full $\boldsymbol{\alpha}^{(\xi)}$ (*full*), Common Component with all diagonal coefficient matrices in the short term component (*c-diag*), Common Component with full $\boldsymbol{\alpha}^{(\xi)}$ (*c-full*).

	$\lvert ret \rvert$			*rkv*			*hl*		
	diag	*full*	*c-diag*	*diag*	*full*	*c-diag*	*diag*	*full*	*c-diag*
full	-0.897			0.358			-1.677		
c-diag	0.126	1.737		-0.259	-0.986		-1.192	0.599	
c-full	0.905	2.190	1.356	-2.810	-3.303	-3.174	-1.319	0.229	-0.207

The medium and bottom panels of Figure 1 illustrate the persistent–transitory decomposition for the three indicators. The persistent components appear fairly smooth, while the transitory components contribute much less to the overall conditional expected volatility, with very little persistence (cf. the $\boldsymbol{\beta}_1^{(\xi)*}$ coefficient). The estimated error term oscillates around one, but is very skewed for absolute returns and fairly symmetrical with occasional spikes (jumps?) for realized kernels. It provides further evidence of the conditional expectation being more accurately estimated for *rkv* (values close to one) relative to *hl* and, even more so, to $\lvert ret \rvert$.

The improved capability of the ACC-vMEM to capture the dynamics of these series is shown in Table 2, where it is clear that a full $\boldsymbol{\alpha}_1^{(\xi)}$ improves the situation over the corresponding diagonal case. The Common Component version (last column) improves matters even more, bringing longer lags into nonsignificant territory.

The results of a Diebold Mariano test (normally distributed test statistics reported in Table 3) show some significant results (negative values indicate that the row model is better than the column model) favoring the Common Component models, especially for the realized kernel volatility, for which the full ACC-vMEM is significantly uniformly better than the other models.

4 Conclusions

Starting from the stylized fact of a high degree of common movements and similar persistence exhibited by time series of several volatility measures (absolute returns, realized kernels and daily range), we suggest a specification of a constrained vMEM ([3], and [4]), forcing its dynamics to follow the evolution of a common component (which all volatility measures contribute to) and of measure–specific components characterized by a lesser degree of persistence. The model can be estimated in a GMM framework on the basis of the expressions for the conditional moments (expectations and variances) of the variables of interest.

In our empirical application the common component is determined mainly by the realized kernel and has a high persistence coefficient as expected; the measure–specific transitory components show a more diversified pattern with a substantial absence of persistence. As expected, the accuracy of the estimated conditional expectation, as measured by the estimated multiplicative errors, is the best for the realized kernels, followed by the daily range and it is much lower for the absolute returns. We notice some occasional spikes in the estimated error, which probably deserve some further attention as occasional bursts in volatility.

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