# A Composite Indicator of Social Inclusion for European Countries

Francesca Giambona and Erasmo Vassallo

**Abstract** Social inclusion is one of seven key challenges of the European Union Sustainable Development Strategy. We use a set of sub-indicators to measure social inclusion for 26 member countries of the European Union. In particular, we aggregate four basic indicators in a multiplicative composite indicator via a DEA-BoD approach with weights determined endogenously by imposing proportion constraints. We obtain a score of social inclusion that allows us to grade 26 European countries in 2008, 2009 and 2010 during the European phase of the financial and economic crisis.

Key words: social inclusion, composite indicator, benefit-of-doubt.

## 1 Introduction

Fighting social exclusion is a key factor for a sustainable development that takes care of resources and aims to increase benefits reducing costs of economic development. The auspices of the European policies for a higher social inclusion of individuals, complementary concept to social exclusion, mainly concern: (a) reduction of monetary poverty; (b) improvement of living conditions; (c) greater access to labour markets; (d) better education (Eurostat, 2011a). Here, we construct a composite indicator of social exclusion/inclusion at macro level aggregating these four pillars represented, each, by a specific basic indicator (Eurostat, 2011b): 1. people at-risk-of-poverty after social transfers (percentage of total population); 2. severely materially deprived people (percentage of total population); 3. people living in households with very low work intensity (percentage of total population); 4. early leavers from education and training (per cent). What weight should be attributed to these indicators? To avoid excessive subjectivity in the choice of the indicators' weights, we use a multiplicative DEA-like

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model in a benefit-of-doubt (BoD) approach. The model is applied to 26 member countries of the European Union: Austria (A), Belgium (B), Bulgaria (BG), Cyprus (CY), Czech Republic (CZ), Denmark (DK), Estonia (EST), Finland (FIN), France (F), Germany (D), Greece (GR), Hungary (H), Ireland (IRL), Italy (I), Latvia (LV), Lithuania (LT), Malta (M), Netherlands (NL), Poland (PL), Portugal (P), Romania (RO), Slovakia (SK), Slovenia (SLO), Spain (E), Sweden (S), United Kingdom (UK); Luxembourg is missing. The composite indicator of social exclusion/inclusion is calculated over the last three years, 2008-2010, during the European phase of the financial and economic crisis. We note that previous years have some data breaks, whereas, at the time of writing, the year 2011 is not available for all countries.

# 2 Multiplicative composite indicator

The weighted product method in construction of the composite indicator of social exclusion (CI) can be formulated as  $CI_i = \prod_{j=1}^n I_{ij}^{w_j}$ , where  $I_{ij}$  is the j-th basic indicator of social exclusion (j=1,...,n) for the i-th country (i=1,...,m) with weight  $w_j$  (OECD, 2008). In particular,  $CI_i$  looks like a geometric mean with some desirable properties, sometimes preferred to the traditional additive approach (Ebert and Welsch, 2004; Zhou and Ang, 2009). In fact, an undesirable feature of additive aggregations is the full compensability they imply. Geometric aggregation is a less compensatory approach, and it offers better incentives to countries to improve underperforming dimensions. Obviously, with multiplicative aggregation, basic indicators must be larger than 1. However, results can depend strongly on the selected weights. To avoid subjective choices easily criticized, in this paper the weights  $w_j$  are determined endogenously with an automatic mechanism based on a multiplicative optimization model similar to a DEA-BoD model written as follows:  $CI_i = \min \prod I_{ij}^{w_i}$ , subject to

 $\prod_{j=1}^{I} I_{ij}^{w_j} \ge e$  with  $w_j \ge 0$  and where e is the Napier's constant (Zhou et al., 2007;

Cherchye et al., 2008; Zhou et al., 2010). Therefore, the composite indicator is obtained by multiplying the four basic indicators with weights calculated to reduce as much as possible the composite score of social exclusion for the *i*-th country. A high value of the composite indicator, i.e. high social exclusion, or rather low social inclusion, is due to high values of the basic indicators and not attributable to specific weights, always calculated to obtain the best, i.e. minimum, possible result for the *i*-th country compared to the benchmark country. In fact, the composite indicator is defined as the ratio of a country's actual performance to its benchmark performance. Searching for the best values, the optimization problem could give zero weight to some indicators and attribute too much weight to other indicators: this is not desirable if all the dimensions are theoretically relevant. Also, this involves no unique ranking and, sometimes, no feasible solution. For these reasons, we add specific constraints on the

weights; in particular, we add proportion constraints: 
$$\left(\prod_{j=1}^n I_y^{w_j}\right)^L \le I_y^{w_j} \le \left(\prod_{j=1}^n I_y^{w_j}\right)^U$$
,

where L and U, ranging between 0 and 1, are the lower and the upper bounds (in percentage terms) for the contribution of the j-th indicator in CI (Cherchye et al., 2008). We set L=20% and U=30%, identical for all the 26 countries and the four basic indicators. Different limits do not give solution to the mathematical problem or lead to an excessive number of benchmark countries or, also, to an unjustifiable imbalance of the indicators' role. The multiplicative optimization problem is nonlinear, then we solve the equivalent linear problem by taking logarithms with base e; at the end, it is easy to obtain the original multiplicative indicator CI. In particular, we refer to scores of social inclusion as SI = e/CI with  $0 < SI \le 1$ : if social inclusion is higher, SI value is higher, where SI = 1 indicates the benchmark country. Finally, we note that in a DEA approach it is not necessary to normalize the basic indicators; conversely, it is necessary to normalize the weights if they are compared because, here, their sum is not

#### 3 Results

Table 1 shows the social inclusion score SI, the corresponding ranking and a mobility index for 26 European countries over the years 2008-2010.

Table 1 Social inclusion score, ranking and mobility index for 26 European countries

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	SI score			Ranking			Mobility	Mobility		SI score			Ranking	Mobility	
	2008	2009	2010	2008	2009	2010	2008-2010		2008	2009	2010	2008	2009	2010	2008-2010
A	0.807	0.875	0.826	11	7	8	1.059	I	0.637	0.687	0.620	26	23	23	0.907
В	0.726	0.750	0.692	16	15	16	0.019	LV	0.731	0.687	0.603	15	24	25	-3.128
BG	0.655	0.665	0.651	23	25	20	0.875	LT	0.872	0.764	0.714	7	13	11	-3.112
CY	0.846	0.912	0.855	9	6	5	0.867	M	0.656	0.696	0.633	22	21	22	0.121
CZ	0.983	1.000	0.998	2	2	2	0.000	NL	0.954	1.000	0.849	5	1	6	-1.373
DK	0.861	0.859	0.765	8	8	10	-0.467	PL	0.883	0.858	0.842	6	9	7	-0.299
EST	0.777	0.795	0.707	12	11	13	-0.376	P	0.654	0.695	0.615	24	22	24	0.062
FIN	0.826	0.851	0.781	10	10	9	0.145	RO	0.638	0.645	0.652	25	26	19	2.170
F	0.771	0.789	0.713	13	12	12	0.067	SK	0.964	0.959	0.900	4	5	4	0.112
D	0.725	0.754	0.700	17	14	15	0.308	SLO	0.972	0.993	0.937	3	3	3	0.000
GR	0.685	0.743	0.673	20	16	18	0.992	E	0.713	0.723	0.598	19	17	26	-4.056
H	0.732	0.701	0.703	14	19	14	-0.058	S	1.000	0.979	1.000	1	4	1	-0.031
IRL	0.719	0.705	0.679	18	18	17	0.087	UK	0.665	0.700	0.643	21	20	21	0.122

Italy occupies the last position in 2008 with a modest improvement in 2009 and 2010, but with increasing distance from the best country (=1-0.637 in 2008 and 1-0.620 in 2010). In fact, Italy has bad performances on the four basic indicators, particularly education and work intensity. On the contrary, Sweden (first position in 2008 and 2010) has excellent performances, especially on monetary poverty and work intensity. It should be noted that the financial and economic crisis has affected the European countries with different intensity and timing, and this helps to explain some strange and strong fluctuations of the countries' positions. For example, from 2008 to 2010, Latvia loses 10 positions (15->25) and Spain loses 7 positions (19->26), but Cyprus gains 4 positions (9->5) and Romania 6 positions (25->19). Of course, if someone loses, there is always someone else who gains; only Czech Republic and Slovenia maintain in all

years the same 2nd and 3rd position, respectively. In general terms, the ability of the 26 European countries to do social inclusion worsens from 2008 to 2010: mean of SI is 0.787 in 2008 and 0.744 in 2010; similarly, median is 0.751 in 2008 and 0.705 in 2010, with minimum 0.637 in 2008 and 0.598 in 2010. In addition, we calculate an index of countries' mobility during the whole period 2008-2010 by combining, after appropriate normalization, the annual difference in rank ( $\Delta R$ ) with the annual difference in score

( $\Delta S$ ) as weight:  $mobility_i = \sum_{i=2000}^{2010} (\Delta R_{i,i}) \cdot (|\Delta S_{i,i}|/\sum_i |\Delta S_{i,i}|) \cdot 100$ . This mobility index has positive (negative) values if, between 2008-2009 and 2009-2010, the *i*-th country has a net improvement (deterioration) of its position. We find that Spain has the worst mobility value (-4.056) and Romania has the best mobility value (2.170). Czech Republic and Slovenia have zero mobility, whereas Italy has a positive mobility value (0.907) (Table 1).

## 4 Conclusion

Social inclusion, considered a key factor for sustainable development, is defined in a multidimensional perspective. Therefore, a summary measure of social inclusion that determines an unique ranking of countries appears of specific interest for policies. In this paper, we obtain a multiplicative composite indicator for 26 member countries of the European Union in the years 2008, 2009 and 2010, by aggregating four basic indicators via a benefit-of-doubt approach with proportion constraints on weights calculated endogenously. We obtain a score of social inclusion *SI*, ranging between 0 and 1, which shows worse performances for Southern Europe, better performances for Northern Europe and a considerable level of social inclusion mobility during the years of the financial and economic crisis.

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