A spatial functional approach for curve classification

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Abstract This paper proposes a spatial functional formulation of the normal mixed effect model for the statistical classification of spatially dependent Gaussian curves. Fixed effect parameters are represented in terms of a functional multiple regression model whose regression operators can change in space. Local spatial homogeneity of these operators is measured in terms of their Hilbert-Schmidt distances, leading to the classification of fixed effect curves in different groups. A real data example in the financial context is analyzed as an illustration.
Abstract *Il presente articolo propone un approccio spazio funzionale per la classificazione di curve. Il modello proposto rappresenta un'estenzione del modello ad effetti misti per dati spazio funzionali. In particolare, paramentri del modello sono rappresentati in termini di una regressione multipla i cui operatori variano nello spazio. Un'applicazione nel contesto finanziario illustra le caratteristiche del metodo proposto.*

Key words: Empirical functional variogram, Firm financial structure, Functional multiple regression, Spatial functional mixed effect models

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1 Introduction

Spatial Functional Statistics (SFS)is emerging as a new branch of Functional Statistics to deal with problems involving spatially dependent curves, or in general, functional data in space. The field of (SFS) still requires further development. The existing literature within this framework covers, among others, statistical methodologies related to spatial functional regression (see [6]), Bayesian inference (see [1]), non-parametric estimation methods (see [2]), spatial functional prediction (see [7], [8], [5], [3]), spatial functional autoregressive time series models (see [12], [13], [15], [14]), outlier detection strategies (see [11]), non supervised clustering methods (see [10], [9], [11]).

However model-based spatial functional statistical classification still remains an unexplored area in most of SFS subfields. This paper propose a spatial functional normal mixed effect approach to classify of spatially dependent curves [16].

Fixed effect curves are represented in terms of a multiple regression model, where the regression operators can change in space. The l^2 distance between the pure point spectra of the non-parametric estimates of these operators (see [15] for derivation of these estimators) allows to detect spatial heterogeneities in the functional linear relationship between the fixed effect curves and the functional regressors. Thus, two fixed-effect curves at different spatial locations are in the same group if the Hilbert-Schmidt distance between the associated vectors of regression operators is sufficiently small according to a prescribed partially data-driven threshold. On the other hand, assuming that the random effect curves display a spatial autoregressive dynamics of order one, the functional variance of their SARH(1) increments is computed for grouping curves with the same mean quadratic spatial local functional variation properties. Note that SARH(1) processes have been introduced in [12] under stationary assumption in space in order to ensure their invertibility. To sum up the proposed functional classification methodology is able to detect the local spatial homogeneity of the regression operators characterizing the fixed effect curves, as well as of the random effect curve increments in the mean-square sense, under a state-space approach. A firm data panel study shows the main steps involved in the implementation of the functional statistical classification methodology proposed in the paper.

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