Generalised Linear Cepstral Models for the Spectrum of a Time Series

Modelli cepstrali generalizzati per lo spettro di una serie storica

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Abstract We introduce the class of generalised linear cepstral models with Box-Cox link, which is based on the truncated Fourier series expansion of the Box-Cox transformation of the spectral density. The link function depends on a power transformation parameter, and encompasses the exponential model. Other important special cases are the inverse link (which leads to modelling the inverse spectrum), and the identity link. One of the merits of this model class is the possibility of nesting alternative spectral estimation methods (autoregressive, exponential, etc.) under the same likelihood-based framework.

Abstract *Il lavoro introduce la classe dei modelli lineari generalizzati cepstrali con link di Box-Cox, che si basa sul troncamento di una base di Fourier per la trasformazione di Box-Cox dello spettro. La classe annida il modello esponenziale e lo spettro inverso. Ciò consente di unificare il trattamento basato sulla verosimiglianza e la selezione di modelli alternativi per la stima dello spettro.*

Key words: Frequency Domain Methods, Generalised linear models, Iteratively reweighted least squares.

1 Introduction

The analysis of stationary processes in the frequency domain has a long tradition in time series analysis; the spectral density provides the decomposition of the total variation of the process into the contribution of periodic components with different frequency as well as a complete characterization of the serial correlation structure of

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the process, so that it contains all the information needed for prediction and interpolation. Inferences on the spectrum are based on the periodogram, which possesses a well established large sample distribution theory that leads to convenient likelihood based estimation and testing methods.

This paper is concerned with a class of generalised linear models formulated for the logarithm of the spectral density of a time series, known as the exponential model, which emerges by truncating the Fourier series expansion of the log-spectrum. The coefficients of the expansion are known as the cepstral coefficients and are in turn obtained from the discrete Fourier transform of the log-spectrum; their collection forms the cepstrum. This terminology was introduced by Bogert, Healy and Tuckey (1963), cepstral and cepstrum being anagrams of spectral and spectrum, respectively.

Bloomfield (1973) introduced the exponential (EXP) model and discussed its maximum likelihood estimation. Local likelihood methods with logarithmic link for spectral estimation have been considered by Fan and Kreutzberger (1998). Also, the exponential model has played an important role in regularized estimation of the spectrum (Wahba, 1980; Pawitan and O'Sullivan, 1994), where smoothness priors are enforced by shrinking higher order cepstral coefficients toward zero, and has been recently considered in the estimation of time-varying spectra (Rosen, Stoffer and Wood, 2009, and Rosen, Wood and Stoffer, 2012).

The paper contributes to the current literature by introducing the class of generalised linear cepstral models with Box-Cox link, according to which a linear model is formulated for the Box-Cox transformation of the spectral density. The link function depends on a power transformation parameter, and encompasses the exponential model, which corresponds to the case when the transformation parameter is equal to zero. Other important special cases are the inverse link (which leads to modelling the inverse spectrum), and the identity link. The coefficients of the model are related to the generalised autocovariances, see Proietti and Luati (2012), and are termed generalised cepstral coefficients. This framework is able to nest alternative spectral estimation methods, in addition to the exponential approach, namely autoregressive spectral estimation (inverse link) and moving average estimation (identity link), so that the appropriate method can be selected in a likelihood based framework.

2 The Exponential Model and Cepstral Analysis

Let $\{y_t\}_{t\in T}$ be a stationary short memory zero-mean stochastic process indexed by a discrete time set T, with covariance function $\gamma_k, k = 0, \pm 1, \ldots,$ and spectral density $f(\omega)$ bounded in $(-\pi, \pi)$.

As $f(\omega)$ is a positive, smooth, even and periodic function of the frequency, its logarithm can be expanded in a Fourier series as follows,

$$\ln[2\pi f(\omega)] = c_0 + 2\sum_{k=1}^{\infty} c_k \cos(k\omega), \tag{1}$$

where the coefficients c_k, $k = 0, 1, \ldots$, are obtained by the (inverse) Fourier transform of $\ln[2\pi f(\omega)]$,

$$c_k = \frac{1}{2\pi}\int_{-\pi}^{\pi} \ln[2\pi f(\omega)]\exp(\imath\omega k)\mathrm{d}\omega.$$

The coefficients c_k are known as the *cepstral coefficients* and the sequence $\{c_k\}_{k=0,1,\ldots}$ is known as the *cepstrum* (Bogert, Healy and Tukey, 1963). The interpretation of the cepstral coefficients as pseudo-autocovariances is also discussed in Bogert, Healy and Tukey (1963) and essentially follows from the analogy with the Fourier pair $2\pi f(\omega) = \gamma_0 + 2\sum_{k=1}^{\infty}\gamma_k\cos(k\omega)$ and $\gamma_k = \int_{-\pi}^{\pi} f(\omega)\exp(\imath\omega k)\mathrm{d}\omega$.

3 The Periodogram and the Whittle likelihood

Due to its sampling properties of the periodogram, a generalised linear model for exponential random variables with logarithmic link can be formulated for the spectral analysis of a time series in the short memory case. The strength of the approach lies in the linearity of the log-spectrum in the cepstral coefficients.

Let $\{y_t, t = 1, 2, \ldots, n\}$ denote a time series, which is assumed to be a sample realisation from a stationary short memory Gaussian process, and let $\omega_j = \frac{2\pi j}{n}$, $j = 1, \ldots, [n/2]$, denote the Fourier frequencies, where $[\cdot]$ denotes the integer part of the argument. The periodogram, or sample spectrum, is defined as

$$I(\omega_j) = \frac{1}{2\pi n}\left|\sum_{t=1}^{n}(y_t - \bar{y})e^{-\imath\omega_j t}\right|^2,$$

where $\bar{y} = n^{-1}\sum_{t=1}^{n} y_t$. In large samples (Brockwell and Davis, 1991, ch. 10)

$$\frac{I(\omega_j)}{f(\omega_j)} \sim \mathrm{IID}\frac{1}{2}\chi_2^2, \qquad \omega_j = \frac{2\pi j}{n},\ j = 1, \ldots, [(n-1)/2], \tag{2}$$

whereas $\frac{I(\omega_j)}{f(\omega_j)} \sim \chi_1^2, \omega_j = 0, \pi$, where χ_m^2 denotes a chi-square random variable with m degrees of freedom, or, equivalently, a Gamma(m/2,2) random variable. As a particular case, $\frac{1}{2}\chi_2^2$ is an exponential random variable with unit mean.

Estimation by maximum likelihood (ML) of the EXP(K) model was proposed by Bloomfield (1971); later Cameron and Turner (1987) showed that ML estimation is carried out by iteratively reweighted least squares (IRLS).

4 Generalized Linear Cepstral Models with Power Link

The generalization that we propose is based on the observation that any continuous monotonic transform of the spectral density function can be expanded as a Fourier

series. We focus, in particular, on a parametric class of link functions, the Box-Cox link (Box and Cox, 1964), depending on a power transformation parameter, that encompasses the EXP model (logarithmic link), as well as the the identity and the inverse links; the latter is also the canonical link for exponentially distributed observations.

Let us thus consider the Box-Cox transform of the spectral density function $2\pi f(\omega)$,

$$g_\lambda(\omega) = \begin{cases} \frac{[2\pi f(\omega)]^\lambda - 1}{\lambda}, & \lambda \neq 0, \\ \ln[2\pi f(\omega)], & \lambda = 0. \end{cases}$$

Its Fourier series expansion, truncated at K, is

$$g_\lambda(\omega) = c_{\lambda,0} + 2\sum_{k=1}^{K} c_{\lambda,k}\cos(\omega k), \tag{3}$$

and the coefficients $c_{\lambda k} = \frac{1}{2\pi}\int_{-\pi}^{\pi} g_\lambda(\omega)\cos(\omega k)\mathrm{d}\omega$ will be named *generalised cepstral coefficients* (GCC).

Hence, a linear model is formulated for $g_\lambda(\omega)$. The spectral model with Box-Cox link and mean function

$$f(\omega) = \begin{cases} \frac{1}{2\pi}\left[1+\lambda g_\lambda(\omega)\right]^{\frac{1}{\lambda}}, & \lambda \neq 0, \\ \frac{1}{2\pi}\exp[g_\lambda(\omega)], & \lambda = 0 \end{cases}$$

will be referred to as a generalised cepstral model (GCM) with parameter λ and order K, GCM(λ, K), in short. The EXP model thus corresponds the the case when the power parameter λ is equal to zero, and $c_{0k} = c_k$, are the usual cepstral coefficients.

For $\lambda \neq 0$, the GCC's are related to the generalised autocovariance function, introduced by Proietti and Luati (2012), $\gamma_{\lambda k} = \frac{1}{2\pi}\int_{-\pi}^{\pi}[2\pi f(\omega)]^\lambda\cos(\omega k)\mathrm{d}\omega$ by the following relationships: $c_{\lambda 0} = \frac{1}{\lambda}(\gamma_{\lambda 0} - 1)$, $\quad c_{\lambda k} = \frac{1}{\lambda}\gamma_{\lambda k}, k \neq 0$. In turn, the generalised autocovariances are interpreted as the traditional autocovariance function of the power-transformed process:

$$u_{\lambda t} = \left[\sigma\varphi\left(B^{\mathrm{S}(\lambda)}\right)\right]^\lambda \xi_t^*, \tag{4}$$

where $\xi_t^* = \sigma^{-1}\xi_t$, $\mathrm{S}(\lambda)$ is the sign of λ, and $\left[\sigma\varphi\left(B^{\mathrm{S}(\lambda)}\right)\right]$ is a series in the lag operator whose coefficients can be derived in a recursive manner based on the Wold coefficients. For $\lambda = 1$, $c_{1k} = \gamma_k$, the autocovariance function of the process is obtained. In the case $\lambda = -1$ and $k \neq 0$, $c_{-1,k} = -\gamma i_k$, where γi_k is the inverse autocovariance of y_t. The intercept $c_{\lambda 0}$ for $\lambda = -1, 0, 1$, is related to important characteristics of the stochastic process, as $1/(1 - c_{-1,0})$ is the interpolation error variance, $\exp(c_{0,0}) = \sigma^2$, the prediction error variance, and $c_{1,0} + 1 = \gamma_0$ is the unconditional variance of y_t. Also, for $\lambda \to 0$, $c_{\lambda k} \to c_k$, i.e. the cepstrum is the limit of the GCC as λ goes to zero.

4.1 Whittle Likelihood Estimation

Let $g_\lambda(\omega_j) = z_j'\theta_\lambda$, with $z_j = [1, 2\cos\omega_j, 2\cos(2\omega_j), \ldots, 2\cos(K\omega_j)]'$ and $\theta_\lambda = [c_{\lambda 0}, c_{\lambda 1}, \ldots, c_{\lambda K}]'$. Then, the approximate Whittle likelihood is

$$\ell(\theta_\lambda) = N \ln 2\pi - \sum_{j=1}^{N} \ell_j(\theta_\lambda),$$

where, for $1 + \lambda z_j'\theta_\lambda > 0$,

$$\ell_j(\theta_\lambda) = \begin{cases} \frac{1}{\lambda}\ln(1+\lambda z_j'\theta_\lambda) + \frac{2\pi I(\omega_j)}{(1+\lambda z_j'\theta_\lambda)^{\frac{1}{\lambda}}}, & \lambda \neq 0, \\ z_j'\theta_0 + \frac{2\pi I(\omega_j)}{\exp(z_j'\theta_0)}, & \lambda = 0 \end{cases}$$

and the approximate Whittle estimator of θ_λ is $\tilde{\theta}_\lambda$ such that

$$\ell(\tilde{\theta}_\lambda) = \max_{\theta_\lambda \in \Theta} \ell(\theta_\lambda),$$

where $\Theta \subseteq \mathbb{R}^{K+1}$.

The score vector and the Hessian matrix, when $\lambda \neq 0$, are respectively

$$s(\theta_\lambda) = \frac{\partial \ell(\theta_\lambda)}{\partial \theta_\lambda} = -\sum_j z_j^* u_j, \quad z_j^* = \frac{z_j}{1+\lambda z_j'\theta_\lambda}, \quad u_j = \begin{cases} 1 - \frac{2\pi I(\omega_j)}{(1+\lambda z_j'\theta_\lambda)^{\frac{1}{\lambda}}}, & \lambda \neq 0, \\ 1 - \frac{2\pi I(\omega_j)}{\exp(z_j'\theta_\lambda)}, & \lambda = 0, \end{cases}$$

$$H(\theta_\lambda) = \frac{\partial^2 \ell(\theta_\lambda)}{\partial \theta_\lambda \partial \theta_\lambda'} = -\sum_j W_j^* z_j^* z_j^{*'},$$

with

$$W_j^* = \begin{cases} \frac{2\pi I(\omega_j)}{(1+\lambda z_j'\theta_\lambda)^{\frac{1}{\lambda}}} - \lambda u_j, & \lambda \neq 0, \\ \frac{2\pi I(\omega_j)}{\exp(z_j'\theta_\lambda)}, & \lambda = 0, \end{cases}$$

so that the expected Fisher information is $\mathscr{I}(\theta) = -\mathrm{E}[H(\theta)] = \sum_j z_j^* z_j^{*'}$.

Maximum likelihood estimation is carried out numerically by the Newton-Raphson algorithm, i.e. iterating until convergence $\tilde{\theta}_{i+1} = \tilde{\theta}_i - [H(\tilde{\theta}_i)]^{-1} s(\tilde{\theta}_i)$ or by the method of scoring: $\tilde{\theta}_{i+1} = \tilde{\theta}_i + [\mathscr{I}(\tilde{\theta}_i)]^{-1} s(\tilde{\theta}_i)$ with fixed initial conditions.

The asymptotic theory for $\tilde{\theta}_\lambda$ is based on Dzhaparidze (1986, ch. 2). For a Gaussian process with positive spectral density function, under the usual regularity conditions (the true parameter θ_0 is an inner point of Θ, the model is identifiable and the derivatives $\frac{\partial f_\theta^{-1}(\omega)}{\partial \theta^{(l)}}$ exist and are continuous for all l), then $\tilde{\theta} \to_p \theta$. If, additionally, $\frac{h' z_j}{[2\pi f_\theta(\omega)]^\lambda} \neq 0$ for all ω and $h = (h_0, \ldots, h_K) \neq 0$ then, setting $z(\omega) = [1, 2\cos(\omega), 2\cos(2\omega), \ldots, 2\cos(K\omega)]'$,

$$\sqrt{n}(\tilde{\theta_\lambda} - \theta_\lambda) \to_d \mathrm{N}(0, V_\lambda), \;\; V_\lambda^{-1} = \frac{1}{4\pi}\int_{-\pi}^{\pi} \frac{1}{[2\pi f(\omega)]^{2\lambda}} z(\omega) z(\omega)' \mathrm{d}\omega.$$

In the exponential case, when $\lambda = 0$, the derivation is the same and so are the asymptotic results, provided that z_j^* is replaced by z_j.

4.2 Reparameterization

The main difficulty with maximum likelihood estimation of the GCM(λ, K) model for $\lambda \neq 0$ is enforcing the condition $1 + \lambda z_j' \theta_\lambda > 0$. This problem is well known in the literature concerning generalised linear models with the inverse link for gamma distributed observations.

The most appropriate solution is to reparameterize the generalised autocovariances and the cepstral coefficients as follows:

$$[2\pi f(\omega)]^\lambda = \sigma_\lambda^2 b_\lambda(e^{-\imath\omega}) b_\lambda(e^{\imath\omega}), \quad b_\lambda(e^{-\imath\omega}) = 1 + b_1 e^{-\imath\omega} + \cdots + b_K e^{-\imath\omega K}, \quad (5)$$

where the b_k coefficients are such that the roots of the polynomial $1 + b_1 z + \cdots + b_K z^K$ lie outside the unit circle, so that, for $\lambda \neq 0$, the GCC's are obtained as

$$c_{\lambda 0} = \frac{1}{\lambda}\left[\sigma_\lambda^2 (1 + b_1^2 + \cdots + b_K^2) - 1\right], \quad c_{\lambda k} = \frac{1}{\lambda}\sigma_\lambda^2 \sum_{j=k}^{K} b_j b_{j-k}.$$

To ensure the positive definiteness and the regularity of the spectral density we adopt a reparameterization due to Barndorff-Nielsen and Schou (1973) and Monahan (1984): given K coefficients $\varsigma_{\lambda k}, k = 1, \ldots, K$, such that $|\varsigma_{\lambda k}| < 1$, the coefficients of the polynomial $b_\lambda(z)$ are obtained from the last iteration of the Durbin-Levinson recursion

$$b_{\lambda j}^{(k)} = b_{\lambda j}^{(k-1)} + \varsigma_{\lambda k} b_{\lambda, k-j}^{(k-1)}, \quad b_{\lambda k}^{(k)} = \varsigma_{\lambda k},$$

for $k = 1, \ldots, K$ and $j = 1, \ldots, k-1$. The coefficients $\varsigma_{\lambda j}$ are in turn obtained as the Fisher inverse transformations of unconstrained real parameters ϑ_j, $j = 1, \ldots, K$, i.e. $\varsigma_{\lambda j} = \frac{\exp(2\vartheta_j) - 1}{\exp(2\vartheta_j) + 1}$ for $j = 1, \ldots, K$, which are estimated unrestrictedly. Also, we set $\vartheta_0 = \ln(\sigma_\lambda^2)$.

By this reparameterisation, alternative spectral estimation methods are nested within the GCM(λ, K) model. In particular, along with the EXP model ($\lambda = 0$), autoregressive estimation of the spectrum arises in the case $\lambda = -1$, whereas $\lambda = 1$ (identity link) amounts to fitting the spectrum of a moving average model of order K to the series. The function that maps the partial autocorrelation coefficients to the model parameters is one to one and smooth (see Barndorff-Nielsen and Schou, 1973, Theorem 2), so that the asymptotic properties of the Whittle estimator continue to hold. The profile likelihood of the GCM(λ, K) as λ varies can be used to select the

spectral model for y_t. A similar idea has been used by Koenker and Yoon (2009) for the selection of the appropriate link function for binomial data; another possibility is to test for the adequacy of a maintained link (e.g. the logarithmic one) using the goodness of link test proposed by Pregibon (1980).

In conclusion, the GCM framework enables the selection of a spectral estimation method in a likelihood based framework. Another possible application of the GCM(λ, K) is the direct estimation of the inverse spectrum and inverse autocorrelations up to the lag K, which arises for $\lambda = -1$ (this corresponds to the inverse link) and of the optimal interpolator (Battaglia, 1983), which is obtained in our case from the corresponding b_k coefficients as $\sum_{k=1}^{K} \rho_{-1,k} y_{t\pm k}$ with

$$\rho_{-1,k} = \frac{\sum_{j=k}^{K} b_{-1,j} b_{-1,j-k}}{\sum_{j=0}^{K} b_{-1,j}^2}.$$

which represents the inverse autocorrelation at lag k of y_t.

5 Illustrations

5.1 Southern Oscillation Index

The Southern Oscillation Index (SOI) measures the difference in surface air pressure between Tahiti and Darwin and it is an important indicator of the strength of El Niño and La Niña events, with values below -8 indicating an El Niño event while positive values above 8 indicate a La Niña event. The index reflects the cyclic warming (negative SOI) and cooling (positive SOI) of the eastern and central Pacific, which affects the sea level pressure at the two locations. The monthly series from January 1876 to December 2013 is plotted in figure 1 along with the autocorrelation function. The series has a periodic behaviour: often the El Niño and La Niña episodes alternate and this confers the SOI a cyclical feature, with an irregular period of about 3-7 years (see e.g. http://earthobservatory.nasa.gov).

We investigate what GCM(λ, K) representation provides the best fit to the sample spectrum of the time series. This depends on two crucial parameters, the truncation parameter K and the power parameter λ, which can be selected by an information criterion, such as the Akaike Information Criterion:

$$AIC(K, \lambda) = -2\ell(\tilde{\theta}_\lambda) + 2K,$$

where $\ell(\tilde{\theta}_\lambda)$ is the Whittle likelihood of the GCM(λ, K) model, evaluated at the maximum likelihood estimate of the parameters θ_λ; the latter is obtained using the reparameterisation considered in section 4.2.

The AIC always leads to $K = 7$ for all values of λ. Figure 2 displays the prediction error variance and the profile Whittle likelihood of GCM(λ, 7) models, as

a function of λ. This suggests that the optimal value of the power transformation parameter is $\tilde{\lambda} = -2.28$.

Fig. 1 Southern Oscillation Index. Time series and sample autocorrelation function. In the first plot the horizontal lines are drawn at ± 8,

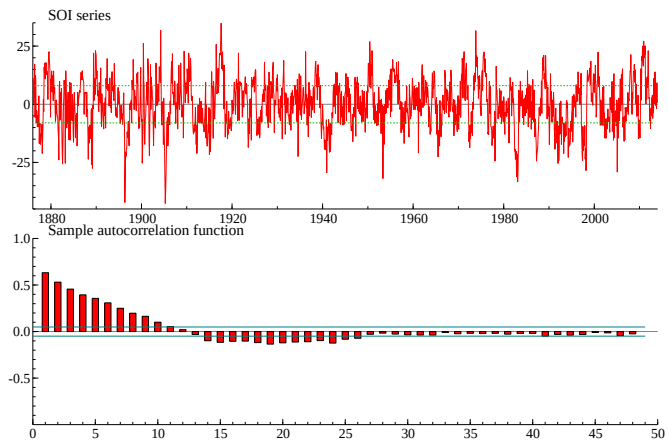
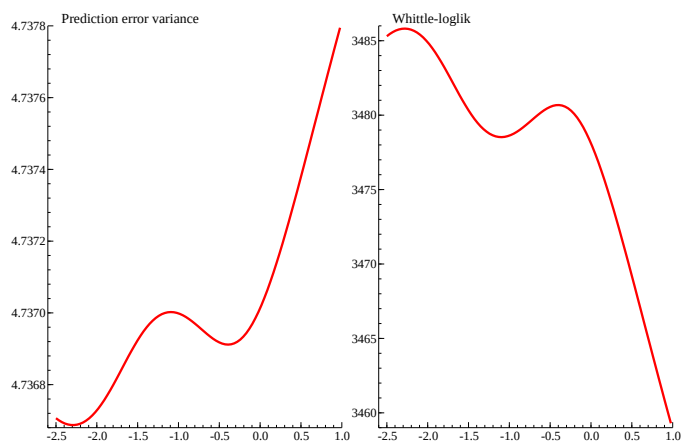


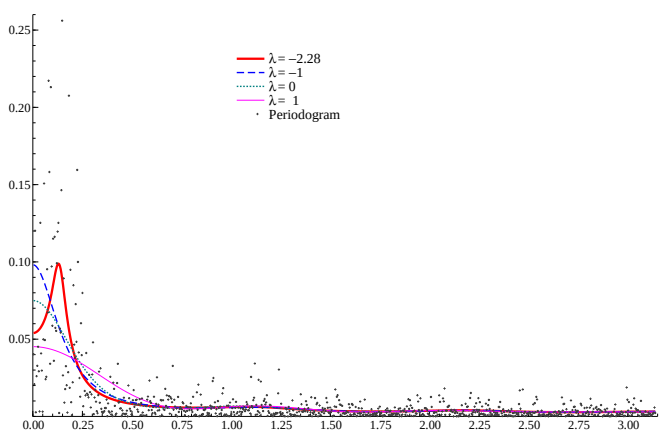
Fig. 2 Southern Oscillation Index. Prediction error variance and Whittle likelihood as a function of λ for GCM(λ, K) models with $K = 7$.



The estimated spectrum is $\tilde{f}(\omega) = \frac{1}{2\pi}\left[\tilde{\sigma}_\lambda^2 \tilde{b}_\lambda(e^{-\iota\omega})\tilde{b}_\lambda(e^{\iota\omega})\right]^{-1/2.278}$, with $\tilde{\sigma}_\lambda^2 = 2161.74$, and the coefficients of $\tilde{b}_\lambda(e^{-\iota\omega})$ are $1, 1.02, 0.03, 0.05, 0.08, -0.04$, 0.02, -0.23. From the second panel of figure 2 it is evident that the likelihood ratio test of $\lambda = -2$ for a GCM(λ, K) model with $K = 7$ is not significant, so that the spectrum that is fitted by maximum likelihood does not differ from that arising from fitting an autoregressive model of order 14 such that the autoregressive polynomial is the square of a polynomial of order 7. This polynomial has three pairs of complex conjugate roots and real root.

Figure 3 plots the periodogram of the SOI series and overimposes the spectral densities fitted fitted by the GCM(λ, K) model with $K = 7$ and λ set equal to 1, 0, -1 and $\tilde{\lambda} = -2.28$. The case when λ is set equal to 1 corresponds to fitting an MA(7) model to the series, whereas the case $\lambda = 0$ corresponds to fitting the EXP(7) model; $\lambda = -1$ corresponds to fitting an AR(7). It should be noticed that in none of these cases a spectral peak arises at a frequency other than zero. The spectrum fitted by maximum likelihood, on the contrary has a clear mode at a frequency corresponding to a period of about four years.

Fig. 3 Southern Oscillation Index. Comparison of the spectral density estimates arising from different GCM(λ, K) models with $K = 7$.



6 Conclusions

Modelling the log-spectrum has a long tradition in the analysis of univariate time series and leads to computationally attractive likelihood based methods. We have devised a general frequency domain estimation framework within which nests the

exponential model for the spectrum as a special case and allows for any power transformation of the spectrum to be modelled, so that alternative spectral fits can be encompassed. As a direction for future research we think that the exponential framework can have successful applications for modelling the time-varying spectrum of a locally stationary processes (Dahlhaus, 2012), by allowing the cepstral coefficients to vary over time.

References

1. Battaglia, F., Inverse Autocovariances and a Measure of Linear Determinism For a Stationary Process, Journal of Time Series Analysis , 4, 79–87 (1983).
2. Barndorff-Nielsen, O.E., Schou, G., On the Parametrization of Autoregressive Models by Partial Autocorrelations, Journal of Multivariate Analysis , 3, 408–419 (1973).
3. Bloomfield, P., An Exponential Model for the Spectrum of a Scalar Time Series, Biometrika , 60, 2, 217–226 (1973).
4. Bogert B.P., Healy M.J.R. and Tukey J.W., The quefrency alanysis of time series for echoes: cepstrum, pseudo-autocovariance, cross-cepstrum, and saphe cracking, in Rosenblatt M. Ed. Proceedings of the Symposium on Time Series Analysis Chapter 15, 209–243. Wiley, New York (1963).
5. Box, G.E.P., and Cox, D.R., An analysis of transformations (with discussion), Journal of the Royal Statistical Society, B , 26, 211–246 (1964).
6. Cameron, M.A., Turner, T.R, Fitting Models to Spectra Using Regression Packages, Journal of the Royal Statistical Society, Series C, Applied Statistics, 36, 1, 47–57 (1987).
7. Dahlhaus, R., Locally Stationary Processes. In *Handbook of Statistics*, Time Series Analysis: Methods and Applications, Volume, 30, chapter 13, Elsevier, p. 351–408 (2012).
8. Dzhaparidze, K., Parameter Estimation and Hypothesis Testing in Spectral Analysis of Stationary Time Series, Springer-Verlag, New York (1986).
9. Fan, J. and Kreutzberger, E., Automatic Local Smoothing for Spectral Density Estimation, Scandinavian Journal of Statistics, 25, 359–369 (1998).
10. Koenker R. and Yoon J., Parametric Links for Binary Choice Models: A Fisherian-Bayesian Colloquy, Journal of Econometrics, 152, 120–130 (2009).
11. McCullagh, T.S. and Nelder, J.A., Generalized Linear Models, Chapman&Hall Cambridge, UK (1989).
12. Monahan, J.F, A Note Enforcing Stationarity in Autoregressive-Moving Average Models, Biometrika, 71, 2, 403–404 (1984).
13. Pawitan, Y. and O'Sullivan, F., Nonparametric Spectral Density Estimation Using Penalized Whittle Likelihood, Journal of the American Statistical Association, 89, 600–610 (1994).
14. Percival, D.B. and Walden, A.T., Spectral Analysis for Physical Applications, Cambridge University Press (1993).
15. Pregibon, D., Goodness of link tests for generalized linear models, Applied Statistics, 29, 15–24 (1980).
16. Proietti, T., and Luati, A., The Generalised Autocovariance Function, MPRA working paper, 47311 (2012).
17. Rosen, O., Stoffer, D.S., Wood, S., Local Spectral Analysis via a Bayesian Mixture of Smoothing Splines, Journal of the American Statistical Association, 104, 249–262 (2009).
18. Rosen, O., Wood, S., Stoffer, D.S., AdaptSPEC: Adaptive Spectral Estimation for Nonstationary Time Series, Journal of the American Statistical Association, 107, 1575–1589 (2012).
19. Wahba, G., Automatic Smoothing of the Log-Periodogram, Journal of the American Statistical Association, 75, 122–132 (1980).
20. Whittle, P., Estimation and Information in Stationary Time Series, Arkiv för Matematik, 2, 423–434 (1953).