A Non-compensatory Composite Index for Comparisons over Time

Matteo Mazziotta and Adriano Pareto

Abstract Most of the socio-economic phenomena such as development, quality of life and well-being have a multidimensional nature and require the definition of a set of individual indicators in order to be properly assessed. Often, individual indicators are summarized and a composite index is created. One of the main problems in constructing composite indices is the choice of a method that allows to assess changes over time. In this paper, we propose a variant of the Mazziotta-Pareto Index (MPI) which allows time comparisons across units to be made. An application to a set of individual indicators of well-being in the Italian regions is presented.

Key words: composite index, ranking

1 Introduction

In the recent years, many have been the attempts of different institutions (universities, statistics offices, international organizations) to construct composite indices of well-being, sustainable development or societal progress [1].

In Italy, since 2010, the National Institute of Statistics (Istat) and the National Council for Economy and Labour (Cnel) have launched a project, called BES (Benessere Equo Sostenibile), for constructing a set of measures of the various dimensions of well-being, at regional level.

An important issue concerning the composite index construction is the level of comparability of the data across units and over time [4]. Comparability of the values of a composite index firstly depends on the normalization rule. All the normalization methods allow for space comparisons, whereas time comparisons may be difficult to make or to interpret.

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Comparisons over time may be absolute or relative. We say that a time comparison is 'relative' when the composite index values, at time *t*, depend on one or more endogenous parameters (e.g., mean and variance of the individual indicators at time *t*). Similarly, we say that a time comparison is 'absolute' when the composite index values, at time *t*, depend on one or more exogenous parameters (e.g., minimum and maximum of the individual indicators fixed by the researcher).

Well-known composite indices of well-being, such as the Index of Regional Quality of Development (QUARS) and the "Il Sole 24 Ore" Quality of Life Index, allow only for relative comparisons since they are based exclusively on values of the individual indicators at the time of reference.

In this paper, we present a variant of the Mazziotta-Pareto Index, namely adjusted MPI, that allows to make absolute comparisons over time, in a not full compensatory perspective [3]. In Sections 2 a brief description of the Adjusted MPI is reported; in Section 3 an application to indicators of well-being in Italy is presented.

2 The adjusted MPI

The Mazziotta-Pareto Index (MPI) is a non-compensatory composite index based on a standardization of the individual indicators, at the reference time, that makes the indicators independent of the variability (normalized indicators have a mean of 100 and a standard deviation of 10) [2]. Therefore, all the individual indicators are assigned equal weights, but only relative time comparisons are allowed.

In order to perform absolute comparison over time, we propose a *re-scaling* of the individual indicators in the range (70; 130) according to two 'goalposts', i.e., a minimum and a maximum value which represent the possible range of each variable for all time periods and for all units.

The steps for computing the adjusted MPI are given below.

Given the matrix X={x_{ij}} with *n* rows (units) and *m* columns (indicators), we calculate the normalized matrix R={r_{ij}} as follow:

$$r_{ij} = \frac{(x_{ij} - \mathrm{Min}_{x_j})}{(\mathrm{Max}_{x_j} - \mathrm{Min}_{x_j})} 60 + 70 \qquad (1)$$

where x_{ij} is the value of the indicator *j* for the unit *i* and Min_{x_j} and Max_{x_j} are the 'goalposts' for the indicator *j*. If the indicator *j* has negative polarity[1], the complement of (1) with respect to 200 is computed.

Denoting with M_{r_i} and S_{r_i}, respectively, the mean and the standard deviation of the normalized values of the unit *i*, the generalized form[2] of the adjusted MPI is given by:

$$\mathrm{MPI}_i^{+/-} = \mathrm{M}_{r_i} \pm \mathrm{S}_{r_i} \mathrm{cv}_i$$

[1] The 'polarity' of a indicator is the sign of the relation between the indicator and the phenomenon to be measured (+ if the individual indicator represents a dimension considered positive and - if it represents a dimension considered negative).

[2] It is a generalized form since it includes 'two indices in one'.

where $cv_i = S_{r_i} / M_{r_i}$ is the coefficient of variation of the unit *i* and the sign ± depends on the kind of phenomenon to be measured.

To facilitate interpretation of results, we suggest to choose the 'goalposts' so that 100 represents a reference value (e.g., the average in a given year).

A simple procedure for setting the 'goalposts' is the following.

Let Min_{x_j} and Max_{x_j} be the overall minimum and maximum of the indicator *j* across all units and all years. Denoting with Ref_{x_j} the reference value for the indicator *j*, the 'goalposts' are defined as: $\mathrm{Ref}_{x_j} \pm \Delta$, where $\Delta = (\mathrm{Max}_{x_j} - \mathrm{Min}_{x_j}) / 2$.[3]

The 'price' to pay for having final scores comparable over time is that individual indicators with different variability are aggregated. However, normalized indicators in an identical range have much more similar variability than original ones.

3 An application to well-being data

In this Section, we show an application of the Adjusted MPI to a number of non-substitutable indicators of well-being in the Italian regions from 2005 to 2011. Variables used are the following: "Life expectancy at birth (years)", "People aged 30-34 with a university degree (%)", "Employment rate for people aged 20-64 (%)", "GDP per capita (Euro)".[4] The adjusted MPI^- is used, since the composite index is 'positive', i.e., increasing values of the index correspond to positive variations of well-being.

Results are reported in Table 1, where 100 represents the Italy's value in 2005 ('goalposts' are calculated by using the procedure described in Section 2).

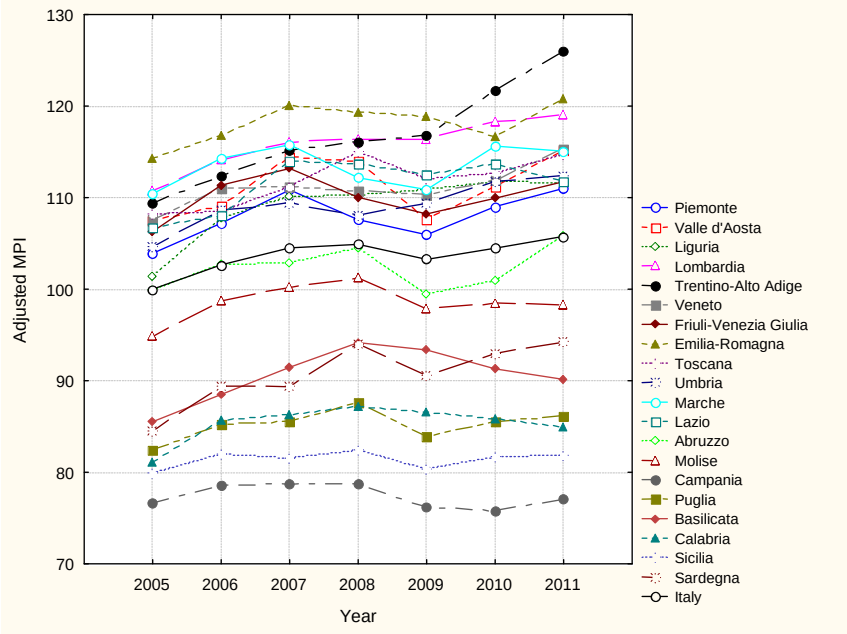
Table 1: Composite Index of well-being in the Italian regions - Years 2005-2011

Region	2005	2006	2007	2008	2009	2010	2011
Piemonte	103.9	107.2	110.8	107.6	106.0	109.0	111.0
Valle d'Aosta	107.5	109.1	114.4	113.9	107.6	111.2	115.2
Liguria	101.5	107.8	110.2	110.3	110.9	111.8	111.6
Lombardia	110.8	114.2	116.0	116.4	116.4	118.3	119.1
Trentino-Alto Adige	109.4	112.4	115.1	116.0	116.9	121.7	126.0
Veneto	107.6	111.0	111.2	110.7	110.4	111.9	115.4
Friuli-Venezia Giulia	106.3	111.4	113.2	110.0	108.2	109.9	111.7
Emilia-Romagna	114.3	116.8	120.1	119.4	118.9	116.6	120.9
Toscana	108.2	108.5	111.2	115.1	112.1	112.7	114.7
Umbria	104.6	108.7	109.5	108.1	109.4	111.8	112.4
Marche	110.4	114.3	115.8	112.2	110.9	115.6	115.1
Lazio	106.7	108.0	114.0	113.7	112.5	113.7	111.7
Abruzzo	99.9	102.7	102.9	104.6	99.5	100.9	105.8
Molise	94.8	98.8	100.2	101.2	97.8	98.5	98.3
Campania	76.7	78.6	78.7	78.8	76.2	75.8	77.1
Puglia	82.5	85.3	85.6	87.6	83.9	85.6	86.2
Basilicata	85.5	88.5	91.5	94.2	93.4	91.3	90.2
Calabria	81.1	85.7	86.3	87.2	86.6	85.8	84.9
Sicilia	80.0	82.0	81.6	82.4	80.4	81.7	81.9
Sardegna	84.5	89.3	89.3	94.0	90.6	93.0	94.3
Italia	**100.0**	**102.7**	**104.5**	**104.9**	**103.3**	**104.5**	**105.8**

[3] Note that the normalized values will fall approximately in the range (70; 130).
[4] Source: http://noi-italia.istat.it

Time series of the composite index are displayed in Figure 1, where a different line pattern and colour is used for each of the regions and referenced in the legend.

Figure 1: Composite Index of well-being in the Italian regions - Years 2005-2011



As we can see, the adjusted MPI allows to compare the trends of the various regions with the national trend (Italy), in contrast to the classic MPI where the average value is set to 100 for each year.

The adjusted MPI may be applied to different domains without loss of comparability. For example, it is possible to compute the composite indices for gender and compare the values obtained with others domains.

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