Implementing Hierarchical Bayesian Model to Fertility Data:the case of Ethiopia

Implementazione di un Modello Gerarchico Bayesiano per dati di Fertilitá: Il caso Dell'Etiopia

Haftu Gebrehiwot and Stefano Mazzuco

Abstract The modeling of fertility patterns is an essential method researchers use to understand world-wide population patterns.Various types of fertility studies have been carried out particularly in developed countries and developing countries outside of Africa to model and look at fertility variations across countries. Nevertheless, much less attention has been given to local or regional fertility curves, where we expect a wider variety of patterns than for country level. Given this variety of possible fertility patterns, in this article, we will propose a model that best captures the different age-specific fertility patterns of Ethiopia at country and regional levels. However, considering the low sample size at local level, a hierarchical Bayesian approach will be considered, in order to have less imprecise estimates.

Abstract *La modellazione di modelli di fecondità è un metodo essenziale usato dai ricercatori per capirne i patterns a livello mondiale. Vari studi sulla fecondità sono stati effettuati in paesi sviluppati e nei paesi in via di sviluppo non africani, per coglierne le differenze. Tuttavia, meno attenzione è stata data alle curve di fecondità a livello locale o regionale, dove ci possiamo aspettare una piÃź ampia variabilità. Considerata questa variabile locale, proponiamo un modello in grado di cogliere al meglio i diversi pattern dei tassi specifici di fecondità dell'Etiopia a livello locale e nazionale. Inoltre, considerata la scarsa numerosità campionaria a livello locale, si utilizzerà un approccio Bayesiano gerarchico, per ottenere stime meno imprecise.*

Key words: ASFR, Fertility Pattern, Fertility rate, Hierarchical Bayesian approach

Haftu Gebrehiwot
University of Padua, Department of Statistical Sciences, Italy, e-mail: gebrehiwot@stat.unipd.it

Stefano Mazzuco
University of Padua, Department of Statistical Sciences, Italy, e-mail: mazzuco@stat.unipd.it

1 Introduction

Ethiopia, a country in East Africa, with 9 ethnically-based administrative regions (a.k.a:National Regional states) (viz.: Tigray, Afar, Amhara, Oromia, Gambella, SNNP, Somali, Benishangul-Gumaz,Harari) and 2 administrative council states (viz.: Addis Ababa, Dire Dawa), as shown in Fig-1 below, is one of the developing countries, attributed, among other things, with high population growth and fertility rates.The country's population in the year 2013 was estimated at 93 million,having increased by almost five-fold in just half a century, from 19.2 million in 1950. This alarming figure in population size rated the country as the most populous nation in Africa next to Nigera and 14th in the world (CIA WorldFactbook,2013). The population growth and fertility rate in the stated year were also 2.9 percent and 4.8 children per woman respectively.This is perceived not to be a good prospect for sustainable development in the country as it is not commensurate with the lower growth rate of agricultural productivity and food production in the country(World Bank, 2013).

Fig. 1: Ethiopia, 9 administrative regions and 2 administrative council states

Like any other developing countries, fertility is one of the most important demographic elements determining the rate of population growth in Ethiopia. In Ethiopia, even though fertility has shown a declining trend at the national level since 2000, the transition is still insignificant in some of the regions. There are also clear and remarkable regional disparities in fertility levels and trends, ranging from 1.5 children per woman (below the replacement level of fertility) in Addis Ababa, where annual growth rate was 2.1, to 7.1 children per woman in Somali region, which had had annual growth rate of 2.6 (CSA and ORC, 2011). It is palpable fact that high fertility rate affects the health status of mothers and the survival chance of their offsprings. Its effects even go to the extent of affecting the socio-economic development of a given country if proper care and action are not taken. Cognizant of this fact, the government of Ethiopia, like most in the developing countries, is perpetrating myriad of efforts to reduce the prevailing high fertility rate to a level the country sets as a target to achieve the Millennium Development Goals (MGDs), which is 4.0 children per woman by 2015. So as to make reaching these targets possible, however, a better understanding of fertility in general and age-specific fertility rates (ASFR) indicators at country and regional levels is of great important.The various reasons can be justified in this regard. Modelling, fitting, parameter estimation and prediction of ASFRs are few among others. Those are worth noting to better make policy decisions, planning, monitoring, and evaluation of programmes in connection to primary education, reproductive health, child health services, population stabilization, human development and other related issues at both the national and regional levels. In recent decades, many studies have been carried out particularly in developed countries and developing countries outside of Africa to model and look at fertility

variations across countries and explore further to understand factors and indicators influencing fertility level. Nevertheless, much less attention has been given to models for local or regional fertility curves, where we expect a wider variety of patterns than for country level. Given this variety of possible fertility patterns, in this article, we will propose a model that best captures the different age-specific fertility patterns of Ethiopia at country and regional levels. However, local estimates might be very imprecise, due to low sample size. We, therefore, make use of a Bayesian hierarchical alternative to the fertility formulas so as to show that the hierarchical model outperforms and is flexible enough for the traditional formulas.In a nutshell, the main objectives of this study are threefold :(1). to examine the pattern of ASFRs at country and regional level;(2). to propose a model that captures various shapes of ASFRs at both country and regional level, and (3). to fit the proposed model using Bayesian Hierarchical techniques.The remaining parts of the paper are structured in the following way: the pattern of ASFRs in Ethiopia is described in the next section. In Section 3, we provide a brief review of some existing models for fertility patterns and then propose our model in Section 4. The results of fitting our model as well as the Bayesian Analyses are presented in Section 5. This paper concludes with a discussion of the implications of our findings.

2 Age Specific Fertility Pattern in Ethiopia: National and Regional Level

The age-specific fertility rate curve, in general, is a bell-shaped unimodal curve which first rises slowly and then sharply in the age group 15-19, attains its modal value somewhere between ages 20-29, declines first slowly and then steeply till it approaches zero around the age of 50 years even though some developing countries has already started showing a deviation from this classical bell shaped curve.Figure 2 below shows the plot of the one-year ASFRs of the 2000, 2005 and 2011 Ethiopian Demography and Health Survey. The plot indicates the age specific fertility patterns are characterized by reciprocal V-shape. Besides, the 2011 pattern confirms that the ASFRs declined in this year much better than in the previous surveys, especially in all of the women younger than age 44. This pattern rises to reach the peak in the late twenties (age group 25-29) before it starts dropping rapidly.This indicates that there was a shift in fertility towards younger age groups in 2011.It is also observed that the highest ASFRs are in the age group 25-29. Therefore, this age group is the most fertile period of Ethiopia women in the stated year. On the other hand, the 2005‘s pattern starts with a relatively higher rate and peaks in the early twenties (age group 20-24). It does not demonstrate a rapid drop after the peak. In contrast to the 2005 and 2011 fertility pattern, the pattern in 2000 depicts a broad peak shape that extends from the early twenties to the early thirties before a rapid drop after the peak. In contrast to the 2005 and 2011 fertility pattern, the pattern in 2000 depicts a broad peak shape that extends from the early twenties to the early thirties before a rapid drop. It is also noticed that the age group 30-35 was the most fertile years in 2000

while 20-24 age group was the most fertile in 2005. Figure 3 also shows the pattern

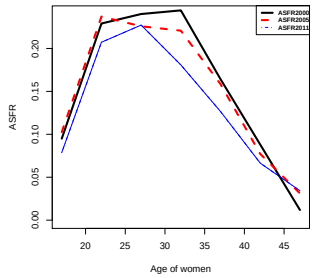

Fig. 2: Trend in ASFRs of Ethiopia in 2000, 2005 and 2011

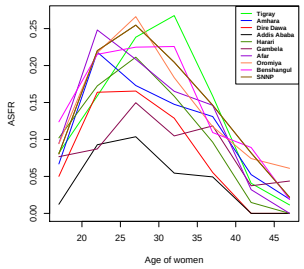

Fig. 3: Pattern ASFRs across the regions,2011 EDHS

of one-year ASFRs at regional level for 2011 EDHS. From the graph, we see that all the age specific schedules for different regions show the traditional pattern except for Somali and Gambella, which could be the effect of small sample size. All in all, considerable variation in the pattern of ASFRs of women is reflected at regions vis-á-vis national level. The figure also revels that ASFRs of Addis Ababa was much smaller than any other regions in the country in all age groups. It also seems that ASFRs of Dire Dawa was the second lowest next to Addis Ababa. In contrary, all other regions have high ASFRs in some age groups and lower in other. Inequalities with respect to the age where the fertility rates reaches a maximum/minimum, and the variable speed with which the maximum is approached from the beginning and is then passed to reach the end of the fertility span are some of the quite orthodox scenarios observed in their patterns.

3 Modelling Fertility at National and regional Levels

A large number of models have been proposed in demographical litratures for modelling the one-year age specific fertility curves of many populations, but fitting these models to curves of Ethiopian data has not been undertaken yet. Among the models applied to ASFR data of different countries are the Coale-Trussell function (Coale and Trussell,1978), the Beta and Gamma(Hoem et al. 1981), the Hadwiger distribution (Hadwiger, 1940), cubic Splines (Hoem and Rennermalm, 1978),the polynomial models(Brass, 1960),the quadratic spline function (Schmertmann, 2003),Model 1 and Model 2(Peristera et al.,2007),and the Adjusted Error Model (Gayawan et al., 2010). To accurately model fertility patterns in Ethiopia, a new mathematical model that is both easily used, and provides good fit for the data is required. Such a model could reveal important parameters which need to be taken

into account when comparing fertility between regions and across time. This undoubtedly would increase our understanding of fertility pattern in the region. In this study, we proposed to use the Skew Normal Distribution to fit the fertility schedules, and show that this proposed model is flexible enough for most fertility patterns at different regions of Ethiopia:

4 Evaluation and Results

4.1 Data

The data applied in this study were obtained from Demographic and Health Survey (DHS), available online from ORC Macro's website (http://www.measuredhs.com). Of the survey data sets available in the site, the 2011, which the latest DHS data set,was chosen to be used in this study. In conclusion, we determined a one-year in stead of three or five-years ASFR data set from the stated survey and hence, will be made use of in the entire work of the analysis.

4.2 Comparison of the proposed model with other ASFR models in Ethiopian context

In order to determine the performance of the proposed model, we conducted some preliminary analysis of fitting the model alongside with ten other commonly used models, namely: the Quadratic spline (QS), Cubic spline (CS), Beta function (BF), Gamma function (GF), Hadwiger function (HF), Skew Normal (SN), Gompertez curve (GC), Adjusred Error Model(AEM), Polynomial function (PF) and Model-1 of Peristera et.al. The purpose of fitting various models is to compare the performance of the proposed model, Skew Normal Distribution, with models mentioned above. The criterion followed in fitting these models is the principle of nonlinear least squares. The **`R`** function **`nls`** was employed for the purpose of fitting these models, estimating parameters, computing some summary measures, and plotting the fitted curves. Results obtained from this preliminary analysis reveal that the values of the AIC for the proposed model, Skew Normal (SN), is lowest for majority, i.e., in 6 of the 11 regions considered. This shows that this model is better able to reproduce the empirical fertility data of Ethiopia and its regions than the other existing models considered.

5 Bayesian Hierarchical Technique

Bayesian statistics has myriad of advantages. One of interesting, important, promising approaches and the focus of current issues is that it deals with fully developed models without constraints on the theoretical assumptions used to develop the models. The idea is to begin with existing theoretically grounded and empirically successful models of fertility, and embed them within a hierarchical Bayesian framework. This embedding opens a vista of potential extensions and improvements to current modeling, because it provides a capability to model the rich structure of fertility in complicated settings.Hence, in the remain work, we will fit the proposed model, the skew normal distribution, using a 2-level Hierarchical Bayesian Approaches taking in to account mothers as a first level and the 11 geographical areas stated above as a second level.

6 Conclusions

In this work , Skew-Normal distribution, which is flexible enough to capture the varying fertility patterns of Ethiopian regions and that has parameters that can be interpreted in terms of demographic indices is presented. The performance of this model alongside some other commonly used models was evaluated using the 2011 Ethiopian DHS fertility data . Using AIC as a model selection criterion, it was determined that the model we propose has the best fit for 6 out of 11 regions. In our application, a 2-level Bayesian hierarchical model will be introduced for parameters estimation and uncertainty analysis.

References

1. Brass, W.:The graduation of fertility distributions by polynomial functions.Population Studies, **14**: 148-162.(1960)
2. Central Statistical Agency (Ethiopia) and ORC Macro: Ethiopia Demographic and Health Survey.Addis Ababa, and Calverton: Central Statistical Agency and ORC Macro.(2011)
3. Coale, A.J. and Trussell, T. J.:Technical note:- finding the two parameters that specify a model schedule of marital fertility. Population Index, **44**,**2**: 203-214(1978)
4. Gayawan,S.B., Ipinyoni, R.A. and Oyejola,B.A.: Modelling fertility curve in Africa.Demographic Research, **22**,211-236(2010)
5. adwiger, H. . Eine analytische reprodutions-funktion fur biologische Gesamtheiten., Skandinavisk Aktuarietidskrift, **23**, 101-113.(1940)
6. Hoem, J. M. and Rennermalm, B.:On the statistical theory of graduation by Splines, University of Copenhagen, Laboratory of Actuarial Mathematics. Working Paper No.**14** (1978)
7. Pasterira,P. and Kastaki, A.: Modelling fertility in modern population.Demographic Research, **16**, 141-194(2007)
8. Schmertmann, C. P.:A system of model fertility schedules with graphically intuitive parameters. Demographic Research,**9**,**5**:82-110(2003)

9. The World Factbook-Central Intelligence Agency.https://www.cia.gov/library/publications/the-world-factbookâĂŐ
10. World Bank : Ethiopia at a glance. http://www.worldbank.org/en/country/ethiopia/overview.(2013)